Article

Non-commutative Geometry & the Holographic Principle

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Abstract

Non-commutative geometries represent position coordinates on a bounding surface of space in terms of non-commuting variables, thereby unifying relativity theory with quantum theory in a natural way. This procedure mathematically formalizes the smallest possible distance scale, called the Planck length, and gives a fundamental explanation for how any possible space-time geometry is quantized. The bounding surface of space is an event horizon that naturally arises as an observer enters into an accelerated frame of reference. The holographic principle is a natural consequence of non-commutative geometries since quantized bits of information are encoded on a bounding surface of space in a pixelated way. The natural pixel size is about a Planck area. This holographic formulation describes whatever appears to happen in the three dimensional space bounded by a two dimensional bounding surface of space. Holography is deeply ingrained in the geometrical nature of relativity theory and the wave-interference nature of quantum theory. No overarching theory is necessary to understand this connection between non-commutative geometries and the holographic principle. An argument is made that an overarching theory is not even possible since such a theory constrains the observer's frame of reference. The principle of equivalence gives the observer the freedom to enter into any possible frame of reference. In a freely falling frame of reference the bounding surface of space disappears. This mechanism, called horizon complementarity, fundamentally connects holography to non-dual metaphysics.

Key Words: non-commutative geometry, holographic principle, horizon complementarity, observer, consciousness.

> *They see only their own shadows, or the shadows of one another, which the fire throws on the opposite wall of the cave. To them, the truth would be literally nothing but the shadows of imagination. See what naturally follows if the prisoners are released and disabused of their error. See the reality of which in his former state he had seen the shadows; and then conceive someone saying to him, that what he saw before was an illusion. His eye is turned towards more real existence, he has a clearer vision.*
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> Plato

Relativity theory is about the nature of space-time geometries that characterize any possible world. Every space-time geometry is characterized by a coordinate system that localizes points in space and instants in time in some world. Such an observable world can only arise in the frame of reference of an observer. In a fundamental way, the observer's frame of reference defines a coordinate system that localizes points in space and moments in time in the observer's world.

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In relativity theory, the observer's frame of reference is called a worldline. The worldline is a path through space and time. Relativity theory tells us that the worldline is a path through space and time followed by an observer, and that the observer is always present at the central point of view of the coordinate system that characterizes that space-time geometry. The observer follows a path through space and time that is at the central point of view of that coordinate system.

The fundamental idea of relativity theory is the principle of equivalence. The basis of this idea is called Mach's principle, which says there is no such thing as an absolute coordinate system that localizes points in space and instants in time. A direct corollary of this idea is there is no such thing as an absolute frame of reference for observations. All observations are relative, and only arise in a frame of reference. Only an observer can enter into a frame of reference and make observations, but all possible frames of reference are relative to each other.

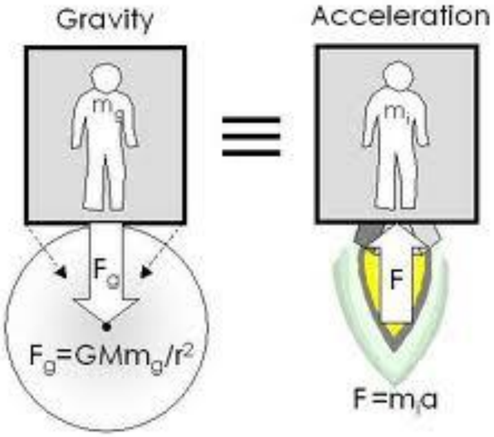
There is a democracy of all possible frames of reference, and the observations made in any possible frame of reference are as valid as those made in any other possible frame of reference. Since there is no such thing as an absolute frame of reference for observations, the observations made in any possible frame of reference are as valid as those made in any other possible frame of reference. Every observer has the inherent validity of its own observations. Relativity theory only describes the relative nature of these differing observations.

Less appreciated than the democracy of all frames of reference is the freedom of the observer to enter into any possible frame of reference. This freedom is a key aspect of the holographic principle. As we'll see when we discuss the implications of horizon complementarity, the observer always has the freedom to enter into a freely falling frame of reference. In a metaphorical sense, an observer that is "attached" to an accelerated frame of reference only has to "let go" in order to enter into a freely falling frame of reference.

Due to the limitation of the speed of light, the democracy of all possible frames of reference leads to some strange phenomena. Even with uniform relative motion, observers in different frames of reference do not observe simultaneous events, nor do they measure the same time intervals. The time intervals measured in different frames of reference appear to become elongated, which is called time dilation. Different observers that move uniformly relative to each other measure different time intervals, and events no longer appear to happen simultaneously.

The space-time geometry arising with uniform relative motion is called Minkowski space. Effects like time dilation are a natural consequence of the uniform relative motion of different observers that arise due to the limitation of the speed of light, which is the maximal rate with which information can be transmitted between different observers. A strange aspect of this maximal rate of information transfer is that all observers observe the same speed of light. This is the fundamental reason observers in different frames of reference observe differing observations.

The situation only becomes more bizarre with accelerated motion. Different observers that are in accelerated frames of reference relative to each other observe the effects of the accelerations. The effects of the accelerations are called forces. Relativity theory expresses this strange state of affairs with the principle of equivalence. This principle states that every observable force is equivalent to the accelerated frame of reference of an observer. In effect, there is no such thing as a force, only the accelerated frame of reference of an observer. The effects of forces only appear to occur in an observer's accelerated frame of reference [2].



The principle of equivalence states every force is equivalent to the accelerated frame of reference of an observer. The reason different observers observe different forces is because they enter into differing frames of reference that move relative to each other with accelerated motion. In effect, there is no such thing as a force, only the differing accelerated frames of reference of observers.

Differing accelerated frames of reference are the origin of curved space-time geometries. A curved space-time geometry arises as an observer enters into an accelerated frame of reference. The apparent force of gravity observed by the observer in that frame of reference is only a consequence of the accelerated motion that characterizes that frame of reference. We can say that the force of gravity is due to the curvature of space-time geometry, but that curvature only arises because the observer is in an accelerated frame of reference.

The other way to describe this strange state of affairs is the observer follows an accelerated worldline through space and time. The observer's accelerated frame of reference gives rise to a coordinate system that localizes points in space and instants in time. The observer is always at the central point of view of that coordinate system and that accelerated frame of reference. The very strange holographic principle tells us this space-time geometry is projected from the observer's holographic screen to the central point of view of the observer.

Although this seems strange, the Kaluza-Klein mechanism tells us an observer also accelerates through extra compactified dimensions of space. The force of gravity is due to accelerations in the extended 3+1 dimensions of space-time. The electromagnetic, strong and weak forces are due to accelerations in the extra compactified dimensions of space-time. Stranger yet, the exponential expansion of space from the big bang event is due to the acceleration of space itself.

An important aspect of accelerated motion is that it explains the nature of energy. Energy does not exist in and of itself, but only arises because an observer enters into an accelerated frame of reference. When we speak about energy, we are really only speaking about another observable aspect of the observer's world.

The expenditure of energy only arises because an observer enters into an accelerated frame of reference. That expenditure of energy is just like a rocket ship that expends energy with the force of its thrusters as it accelerates through space. That exerted force is equivalent to the accelerated frame of reference of the observer. Relativity theory quantitatively specifies that kinetic energy arises with all motion due to the effects of time dilation, while potential energy arises in an accelerated frame of reference, which is equivalent to the exertion of a force.

Just like all the observable things in the observer's world, the expenditure of energy within that world does not have its own independent objective existence, but only appears to exist because the observer enters into an accelerated frame of reference. The observation of the expenditure of energy can only appear to occur in the observer's frame of reference.

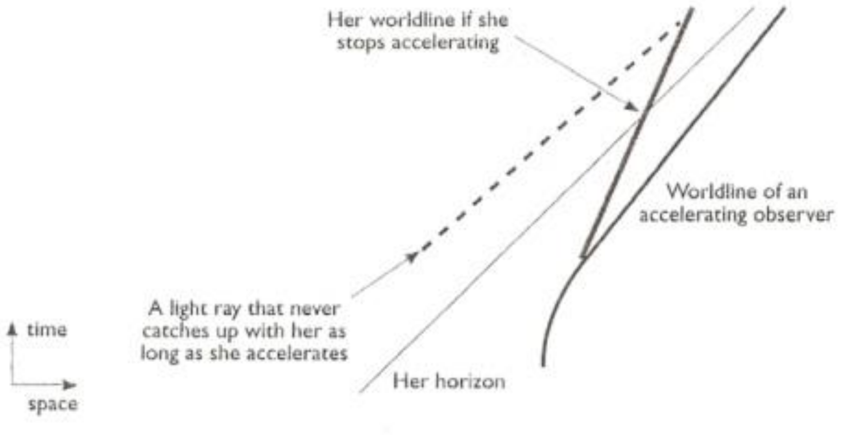
The observer not only observes the form of the observable things, but also the motion of the observable things, which is the nature of kinetic energy, and the way the observable things interact with each other through the effect of forces, which is the nature of potential energy. The observation of the expenditure of energy, whether expressed by the apparent motion of things or by the apparent forceful interactions between things, only arises because the observer follows an accelerated worldline and enters into an accelerated frame of reference.

An observer's accelerated frame of reference not only gives rise to observations of apparent forces, but to other strange effects. The strangest of these effects is the observation of an event horizon. An event horizon is a boundary in space that limits the observer's observations in space. This limitation is due to the limitation of the speed of light. An event horizon demarcates a boundary in space that is as far out in space as the observer at the central point of view of that space-time geometry can see things in space due to the limitation of the speed of light.

Event horizons naturally arise in the curved space-time geometries that arise as an observer enters into an accelerated frame of reference. This occurs with any kind of accelerated motion [3]. The most famous examples of event horizons are black hole horizons that arise due to the force of gravity and cosmic horizons that arise with the exponential expansion of space. A Rindler horizon arises with any kind of accelerated motion. As we'll see, all of these horizons have a temperature and radiate away Hawking radiation as a form of thermal radiation. These examples will be discussed in detail in due course.

As an observer enters into an accelerated frame of reference, an event horizon arises that limits the observer's observations in space. The horizon demarcates a boundary in space that is as far out in space as the observer at the central point of view can see things in space. The event horizon is only a bounding surface of space surrounding the observer that limits observations due to the limitation of the speed of light. No light signal that originates beyond the horizon can ever reach the observer as long as the observer follows an accelerated worldline and remains in an accelerated frame of reference. Since nothing can travel faster than the speed of light, nothing is observable beyond the horizon.

Event horizons arise with any kind of accelerated motion. As an observer enters into an accelerated frame of reference and follows an accelerated worldline, an event horizon arises that limits the observer's observations in space. If the observer stops accelerating and enters into a freely falling frame of reference, the observer's horizon disappears and its observations in space are no longer limited [3].



The key idea that allows for unification of relativity theory with quantum theory is the observer's frame of reference. Only an observer can enter into a frame of reference and follow a worldline through space and time. Only an observer can make observations within that frame of reference. The key concept that allows for unification of relativity theory with quantum theory is that the observer must exist before it enters into any possible frame of reference and before the observation of anything that appears to happen within that frame of reference is possible. This key idea is important enough that it bears repeating:

Before the observation of anything is possible, the observer must exist.

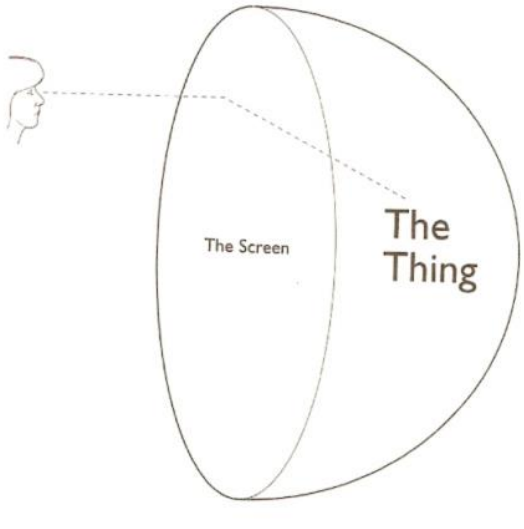
The observer must have an independent existence that pre-exists anything that appears to come into existence as the observer enters into a frame of reference. The apparent existence of all observable things is observer-dependent, and only appear to come into existence as the observer enters into a frame of reference. Not only the apparent form of the observable things comes into existence, but also the apparent motion of the observable things and the apparent interactions between the observable things comes into existence. Both the apparent form of things and the apparent expenditure of energy that animates those things comes into existence as the observer enters into an accelerated frame of reference.

How is this possible? The answer is the holographic principle [4], which is a direct consequence of the unification of relativity theory with quantum theory. As an observer enters into an accelerated frame of reference, an event horizon arises that limits observations in space due to the limitation of the speed of light. The event horizon is a bounding surface of space that surrounds the observer present at the central point of view and limits the observer's observations in that bounded space. Everything the observer observes is limited within that bounded space.

The observer's horizon is a bounding surface that surrounds the observer at the central point of view, and limits the observer's observations to whatever appears to happen within that bounded space. This is due to the limitation of the speed of light, which is the maximal rate of information transfer. The observer's horizon only arises because the observer is in an accelerated frame of reference. Anything that appears to happen beyond the observer's horizon is unobservable to the observer since a signal about what appears to happen there would have to travel faster than the speed of light to ever reach the observer. That is impossible since nothing can travel faster than the speed of light and no signal can be transmitted faster than the maximal rate of information transfer between different observers.

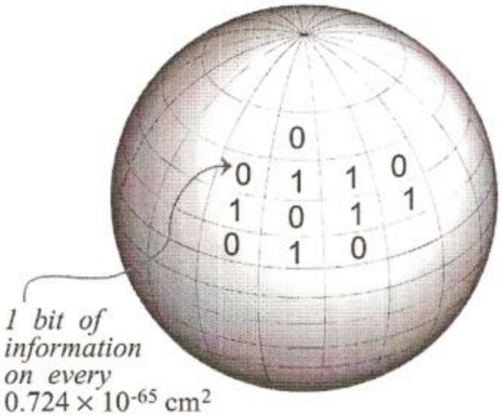
The holographic principle is deeply ingrained in the geometric nature of relativity theory and the wave-interference nature of quantum theory, and so arises with the unification of relativity theory with quantum theory. The geometrical aspects of the holographic principle are very general focusing theorems [5] in relativity theory. Focusing theorems tell us that light rays emitted orthogonal to the surface of an event horizon can always become focused at a focal point. That focal point localizes the central point of view of an observer that follows a worldline through space-time. The event horizon is a bounding surface of space that surrounds the observer that enters into an accelerated frame of reference. The encoding of bits of information on the event horizon is due to the wave-interference nature of quantum theory. This is best explained with a non-commutative geometry. Non-commuting position coordinates defined on a bounding surface give rise to the encoding of bits of information due to uncertainty in their measurements.

The holographic principle tells us the world we appear to live within does not really have three spatial dimensions, but is only two dimensional. This insight is due to a more fundamental understanding of the nature of quantum information. All the fundamental quantized bits of information for anything we can perceive or that appears to happen in any three dimensional region of space are encoded on the two dimensional bounding surface of that space. The encoding of information on a two dimensional surface for all the things perceived in a three dimensional region of space is the nature of a hologram. Perceivable things are like images projected from a holographic screen to the central point of view of an observer [3].



The holographic principle tells us the bounding surface of space acts as a holographic screen and encodes all the information for everything observed in that bounded space. Everything observed within that bounded space is like the projection of animated images from the holographic screen to the central point of view of the observer. Just like a movie displayed on a digital computer screen, the images are animated over a sequence of screen outputs or observational events that arise in the flow of energy. Due to the magic of holography, the observable images appear three dimensional even though all the information for the images is encoded on the two dimensional bounding surface of that space.

To be clear about things, the holographic principle tells us that all the quantized bits of information for the perceivable three dimensional images of anything we can perceive in any three dimensional region of space are encoded on the two dimensional bounding surface of that space. That surface acts as a holographic screen that encodes information in a pixelated way, with one bit of information encoded per pixel on the screen. Information is fundamentally encoded in a binary code of 1's and 0's, like in a computer. The pixel size is determined by the fundamental distance scale called the Planck length, which is about 10^{-33} cm, and determines the size of a pixel as about 10^{-65} cm^2, or as a Planck area [6].



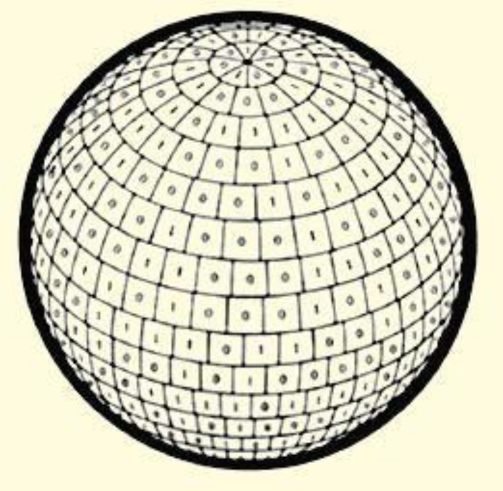
Why is the holographic principle the natural consequence of the unification of relativity theory with quantum theory? The answer has to do with the nature of quantum uncertainty and quantum information. The holographic principle arises from a more fundamental understanding of the uncertainty principle as it applies to the measurement of position coordinates in space.

The holographic principle has to do with the way in which space-time geometry is quantized, and is a fundamental aspect of any theory of quantum gravity. To fully understand the holographic principle requires a more fundamental understanding of the uncertainty principle as it applies to the space-time geometry that arises in the observer's accelerated frame of reference. This has to do with how points in space and instants of time are quantized. This understanding has come about in recent years with a better understanding of non-commutative geometries. Although this subject may seem complex, non-commutative geometries can be understood intuitively, and give a fundamental explanation for how information is encoded on a bounding surface of space that acts as a holographic screen.

The basic problem is there is no such thing as an absolute frame of reference for observations, and so there is no such thing as an absolute coordinate system that can characterize space. All observations must occur in the relative frame of reference of an observer. Only an observer can enter into a frame of reference. Every possible coordinate system is a possible frame of reference that is relative to all other possible frames of reference, and an observer is always present at the central point of view of each coordinate system.

The more fundamental way of understanding quantum uncertainty is in terms of the uncertainty in the position coordinates that define any coordinate system that attempts to characterize space. Since space cannot be characterized in this way, there is always some uncertainty in those position coordinates. The easiest way to understand this is in terms of the position coordinates that characterize a bounding surface of space like an event horizon. If that bounding surface of space is a sphere, then any point on the event horizon is defined by a set of latitude and longitude coordinates, which we can call (x, y).

Since there is no such thing as an absolute coordinate system defining space, there is always some uncertainty in these position coordinates. The amount of uncertainty is called the Planck length. The more accurately we know about the x-coordinate, the less accurately we know about the y-coordinate. The result of this uncertainty in position is the (x, y) point on the surface of the event horizon gets smeared out into an area element the size of a Planck area. That area element acts like a pixel on the screen and encodes a bit of information [7].



In quantum theory, we understand the uncertainty principle in terms of quantum operators. For example, if we have a particle located at a position x that moves with a momentum p, we represent the momentum operator as $p=-i\hbar\partial/\partial x$, and so $px-xp=-i\hbar$. This allows us to write an eigenvalue equation for the wave function, $-i\hbar\partial\psi/\partial x=p\psi$, which has a solution $\psi(x)=\exp(ipx/\hbar)$. This wave function implies an uncertainty relation in terms of simultaneous measurements of position and momentum, which is usually written as $\Delta x\Delta p\geq\hbar$. The more accurately we know about the position x, the less accurately we know about the momentum p.

The same kind of quantum operators and uncertainty relation applies to a non-commutative geometry, except these are position operators defined on a bounding surface of space, like an event horizon. If that surface is a sphere, we can locate positions on the sphere in terms of latitude and longitude, which we can call (x, y). Position operators are written in terms of Dirac operators [8], and so have internal degrees of freedom, like spin. The position coordinates (x, y) of the surface are represented by these non-commuting quantum operators.

The position operators in a non-commutative geometry do not represent the uncertainty in the position and momentum of a particle, but rather represent the uncertainty in the position coordinates of space itself. This uncertainty relation usually takes the form $\Delta x \Delta y \geq \ell^2$, where ℓ is the Planck length. The more accurately we know about the x-coordinate, the less accurately we know about the y-coordinate.

A non-commutative geometry gives a fundamental explanation for how any possible space-time geometry is quantized. This inherently involves uncertainty in the measurement of position coordinates of space at any point in space. On the surface of a sphere, a point is represented as (x, y), but the uncertainty relation $\Delta x \Delta y \geq \ell^2$ implies that the point is smeared out into a Planck size area element. The more accurately we know about x-coordinate, the less accurately we know about the y-coordinate, and so we can never really measure the (x, y) point. Like anything else in quantum theory, non-commuting position coordinates defined on a bounding surface of space have inherent uncertainty in their measurements.

It helps to back up before we discuss non-commutative geometry further, and review the ordinary quantum theory of particles. The uncertainty principle is fundamental to the formulation of quantum theory. If we want to measure the location of a particle in space, we usually shine light at the particle and observe the pattern of scattered radiation. We can think of light as a wave, but quantum theory tells us each quantum of light acts like a particle called a photon. When a photon interacts with another particle in a scattering event, momentum is always exchanged between the particles. The photon has an intrinsic wavelength related to its momentum as $p=h/\lambda$. This wavelength limits the degree to which we can localize the particle's position in space. If we want to localize the particle within a distance interval of $\Delta x=\lambda$, an amount of momentum at least as large as $\Delta p=h/\Delta x$ must be exchanged in the scattering event, which is the uncertainty principle.

Particles not only scatter off each other in scattering events, but they also form bound states, like the bound state of an electron and a proton in a hydrogen atom. Bound states occur with the attractive electromagnetic force between positively charged protons and negatively charged electrons. Bound states are quantized into discrete periodic orbits due to the nature of wave functions. As a bound state of an electron and a proton forms, the electron's wave function is constrained to fit into the circumference of a periodic orbit. We call this constraint on the wave function the requirement of periodicity in a compactified space.

Since momentum is related to wavelength as $p=h/\lambda$, this constrains the particle's momentum in terms of the number of wavelengths that can fit into the orbit. A quantum number n naturally arises from the number of wavelengths that can fit into a periodic orbit. For a circular orbit of radius r and circumference $2\pi r$, the wavelength is constrained as $n\lambda=2\pi r$, resulting in quantized values of momentum $p=n\hbar/r$.

The quantized values of particle momentum that arise in bound states also give rise to quantized energy levels. Total energy is only a sum of kinetic and potential energy $E=KE+PE$. For a circular orbit, kinetic energy is constrained in terms of the orbital radius since momentum is constrained in this way. In a hydrogen atom the electron's kinetic energy depends on its momentum $p=mv$ as $KE=p^2/2m$, but its momentum is constrained in terms of its radius as $p=n\hbar/r$. The electron's potential energy depends on its radius as $PE=-e^2/r$, where $-e$ is the charge of the electron. Since both momentum and potential energy depend on the radius, total energy has an effective form that only depends on the orbital radius.

Quantized energy levels arise from the requirement of minimum energy. The effective total energy of each possible orbit depends on the radius of that orbit, and always has a minimum value as a function of the orbital radius. Minimum energy not only determines the quantized energy levels, but also the orbital radius of the quantized energy levels. In the sense of the action principle, a bound state orbit is a path through space and time that minimizes energy.

Bound state energy levels all have negative energy, since the potential energy of an attractive force has a negative value. If total energy is negative, a bound state is forced to form, but is constrained in terms of quantized energy levels. If total energy is positive, only unbound scattering states can arise, but scattering states are also quantized. The state of zero total energy, $E=0$, is called escape velocity. A particle with escape velocity has just enough kinetic energy to escape away from an attractive force that gives rise to the bound state.

As a charged particle, like an electron, makes a transition from one bound state energy level to another energy level, a quantum of electromagnetic radiation is absorbed or emitted. This quantum of electromagnetic radiation is called a photon, and carries a quantized amount of energy $E=hf$ determined in terms of its wave frequency.

This discussion of bound states is relevant for how quantized bits of information arise on a bounding surface of space. The best example to consider is the bound state of a black hole that arises due to the force of gravity. The event horizon of the black hole is a bounding surface of space that defines this bound state.

An easy way to determine the radius of a black hole event horizon is to use the concept of escape velocity. A particle of mass m that moves with velocity v has a kinetic energy $KE=\frac{1}{2}mv^2$. As that particle moves in the gravitational field of a black hole of mass M and at a distance R from the center of the black hole, it experiences a potential energy $PE=-GmM/R$. The minus sign indicates that gravity is an attractive force.

The nature of forces is that potential energy can be converted into kinetic energy, like a particle that picks up speed due to the force of gravity as it falls down a hill, and loses speed as it climbs the hill. When a particle has escape velocity, it has just enough kinetic energy to escape away from the attractive force. Escape velocity is thus determined by $E=KE+PE=0$.

The event horizon demarcates a boundary in space where even light cannot escape away. For a black hole, escape velocity is the speed of light $v=c$, which gives the radius of the event horizon as $R=2GM/c^2$. Although this simple calculation of the horizon radius relies on concepts from non-relativistic physics, a relativistic calculation gives the same answer. The correct way to calculate the radius is to first find the Schwarzschild solution of Einstein's equations and then determine the radius from the metric $ds^2=(1-2GM/c^2r)dt^2-(1-2GM/c^2r)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2)$, but the idea of escape velocity is more intuitively obvious [9].

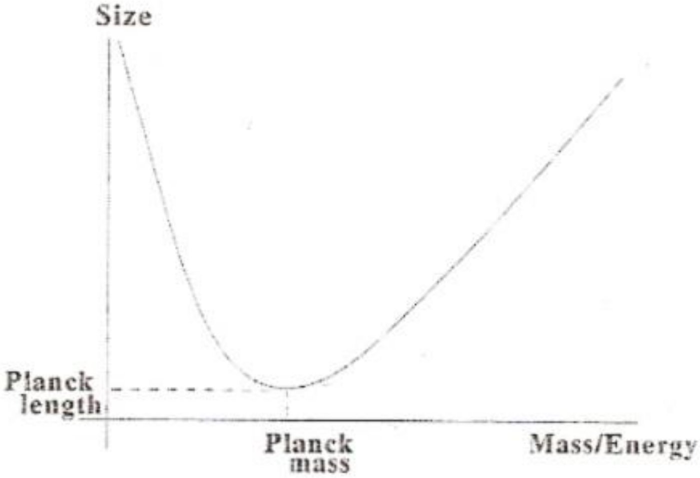
The event horizon of a black hole is a bounding surface of space that limits an observer's observations in space due to the limitation of the speed of light. The event horizon is a surface that demarcates a boundary where the force of gravity is so strong that even light cannot escape away from the black hole. For an external observer, no signal that originates within the event horizon can ever cross the horizon due to the force of gravity and reach the external observer. The radius of the event horizon is directly related to the amount of mass stored within the black hole as $R=2GM/c^2$. If enough mass and energy is concentrated in a small enough region of space, a black hole is forced to form due to the force of gravity, which is so strong at the event horizon that even light cannot escape.

This result tells us something very odd about how we measure the location of a particle by scattering electromagnetic radiation off the particle. Photon energy is related to wave frequency as $E=hf$, which is the same as $E=hc/\lambda$, since the speed of light is given by $c=\lambda f$. This means if we want to localize the particle's position within a smaller distance scale $\Delta x=\lambda$, we're forced to use higher energy photons. Eventually, as we measure distances at smaller and smaller distance scales, we concentrate so much energy into such a small region of space that a black hole is forced to form. If we equate this amount of energy with the mass of the black hole, as $E=hc/\lambda=Mc^2$, and look for the smallest possible black hole radius as $R=\ell$, this determines the radius of the smallest possible black hole as $\ell=2hG/\lambda c^3$. If the wavelength at which a black hole is forced to form is related to the smallest possible black hole radius as $\lambda=4\pi\ell$, similar to how a wavelength is constrained to fit into the circumference of a periodic orbit, then we determine the smallest possible distance scale in terms of the Planck length $\ell^2=\hbar G/c^3$.

This is a very odd result. We measure the location of the particle by scattering radiation off the particle, but at a very high energy level and very small distance scale a black hole is forced to form with a radius $R=\ell$. This smallest possible distance scale is the Planck length.

As we shine higher energy photons with a smaller wavelength at this black hole, we only concentrate more energy within the black hole, which means the black hole gains a larger mass and its radius increases. The photons we measure that are radiated away from the surface of the black hole have a wavelength that is approximately related to the radius of the event horizon as $\lambda=4\pi R$, which is about the maximal circumference of the horizon.

As we shine higher energy photons at the black hole we create a more massive black hole with a larger radius, which can only radiate smaller energy photons back to us with a larger wavelength. The distance scale at which the black hole is forced to form $R=\ell$ is the smallest distance scale that we can measure. If we try to measure anything smaller than a Planck length, that thing appears to become larger, since we have forced a black hole to form. The Planck length is the smallest possible distance scale that we can ever measure. If we try to measure anything smaller, we only force a black hole to form, and that thing appears to become larger. This is very weird, but that is how quantum gravity works [4].



A non-commutative geometry is a way of mathematically formalizing this smallest possible distance scale. The easiest example to consider is the event horizon of a black hole. The event horizon is a spherical surface of radius R. Just like with quantum theory, a non-commutative geometry is characterized by an uncertainty relationship, but this relationship is not between the positions in space and the momentums through space of particles. This uncertainty relationship is between the position coordinates of space itself. In a non-commutative geometry there is always some uncertainty in our measurement of the position coordinates of space.

The reason for this strange state of affairs is the absolute nature of empty space. We try to do something impossible as soon as we set up a coordinate system and try to define the nature of space, since the absolute nature of space cannot be defined. The absolute nature of space cannot be defined, characterized or limited in any possible way. The absolute nature of space is only like an empty space of potentiality that gives rise to all the definable aspects of all observable things. As pure potentiality, empty space cannot be defined. Only observable things as characterized by the observable values that arise in a quantum state of potentiality can be defined. These are the observable things we observe in the world. As the source of the quantum state of potentiality for any possible world, the absolute nature of empty space cannot be defined, not even in terms of its dimensionality. As soon as we try to set up a coordinate system in space, we try to define space, but the absolute nature of empty space cannot be defined.

This odd state of affairs is absolutely required if we are to give relativity theory its correct interpretation. Only the coordinate systems that arise in relative frames of reference are definable, but those relative frames of reference are all equivalent to each other due to the equivalence principle. The relative frames of reference can appear to move relative to each other, and if that motion is accelerated, different forces can appear in the different frames of reference. Each possible frame of reference is characterized by its own coordinate system, and an observer is always present at the origin or central point of view of every frame of reference.

Every possible frame of reference is a frame of reference for observations. All observations occur in a frame of reference, but those observations are relative to all other possible frames of reference. Only an observer in a relative frame of reference can make observations. There is no such thing as an absolute frame of reference for observations, and so there is no such thing as an absolute coordinate system. The impossibility in relativity theory of an absolute coordinate system or an absolute frame of reference for observations is called Mach's principle.

A non-commutative geometry solves this problem by specifying that all possible positions in space as defined by a coordinate system have inherent uncertainty. This is accomplished by representing the position coordinates on a surface with non-commuting operators. If we consider the event horizon of a black hole, that two dimensional surface can be parameterized in terms of an x-y coordinate system labeled by points on the surface as (x, y). On the surface of a sphere these coordinates are like latitude and longitude. A non-commutative geometry specifies there is inherent uncertainty in our ability to measure the (x, y) points.

A non-commutative geometry only describes the inherent uncertainty in our ability to measure points in space as defined on a bounding surface of space. Quantum theory tells us that we are no more able to measure points in space with certainty than anything else we can measure.

A bounding surface of space arises as an observer enters into an accelerated frame of reference. We call a bounding surface of space an event horizon. A non-commutative geometry tells us the position coordinates defining points on a bounding surface are represented by non-commuting variables, and so their measurement has inherent uncertainty. Like any other non-commuting variable in quantum theory, there is inherent uncertainty in our ability to measure a position coordinate on a bounding surface of space. Ordinary quantum theory describes the uncertainty in the measurement of position and momentum of particles, while a non-commutative geometry describes uncertainty in the measurement of the position coordinates of space itself. This uncertainty is the reason a bounding surface acts like a holographic screen that encodes bits of information on pixels.

This uncertainty relation is usually expressed as $\Delta x \Delta y \geq \ell^2$. With a measurement, the more accurately we know about the x-coordinate the less accurately we know about the y-coordinate. It is as though the (x, y) point on the surface of the sphere gets smeared out into an area element of size ℓ^2. An area element acts like a pixel on the surface and encodes a bit of information.

The number of bits of information encoded on the surface depends on the number of pixels. If only two bits of information are encoded, there are only two pixels. In a non-commutative geometry this is represented by a 2x2 matrix. Information is encoded in terms of the eigenvalues of the matrix. A 2x2 matrix has two eigenvalues and thus can encode two bits of information. In quantum theory a 2x2 matrix can represent a spin ½ variable. A spin ½ variable only has two measurable values, up or down, and acts like a switch that is either on or off. A spin ½ variable can thus encode information in a binary code of 1's and 0's. If n pixels are defined on the surface of the sphere, then n bits of information are encoded, represented in a non-commutative geometry by an nxn matrix.

The origin of nxn matrices in a non-commutative geometry are non-commuting position coordinates, which are represented by Dirac operators. Since the Dirac operator is defined in terms of γ-matrices that generically are nxn matrices that obey an anti-commuting relationship of the form $\frac{1}{2}[\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu]=g_{\mu\nu}$, where $g_{\mu\nu}$ is the Minkowski metric with signature (+, –, …, –), the nxn matrices that localize positions on the surface of the sphere are SU(n) matrices. The n eigenvalues of an SU(n) matrix can then encode n bits of information. The symmetry inherent in this procedure allows an SU(n) matrix to localize n position coordinates on the sphere in a rotationally invariant way, but the position coordinates all have inherent uncertainty.

An nxn matrix is always decomposable into 2x2 matrices, and so can encode n bits of information in a binary code, in much the same way that spin ½ variables encode information in a binary code in terms of the eigenvalues of the matrix. In the sense of quantum entanglement, these bits of information are entangled due to the nature of matrices. If the pixel size is $4\ell^2$, a spherical surface of radius R and surface area $A=4\pi R^2$ can encode $n=A/4\ell^2$ bits of information.

A key aspect of the holographic principle and of a non-commutative geometry is that it makes no sense to speak of distances less than the Planck length, since this is the smallest possible distance scale that can be measured. Uncertainty in all position coordinates smears out any point in space into a Planck size element and makes the point appear fuzzy. It makes no sense to discuss distances smaller than this smallest possible measurable distance scale.

A non-commutative geometry gives a natural explanation for the holographic principle and how bits of information are encoded on any bounding surface of space. The fundamental reason for non-commutative geometries is the absolute nature of space cannot be defined in terms of an absolute coordinate system, which is to say there is no such thing as an absolute frame of reference for observations.

All coordinate systems are relative to each other and arise in the relative frame of reference of an observer. Only an observer can enter into a frame of reference and make observations. As an observer enters into an accelerated frame of reference, an event horizon arises that surrounds the observer at the central point of view. A non-commutative geometry gives a natural explanation for how bits of information are encoded on that bounding surface of space.

The fundamental way this principle is mathematically expressed is in terms of an uncertainty relation that specifies the inherent uncertainty with which any position coordinate defined on the bounding surface of space can be measured, $\Delta x\Delta y\geq\ell^2$. This uncertainty smears out the (x, y) coordinate into an area element like a pixel, and each pixel encodes a bit of information. The pixels are always defined on a bounding surface of space. A bounding surface of space is only an event horizon that arises in the accelerated frame of reference of an observer, but it limits observations within that space and acts as a holographic screen.

A quantum state of potentiality is about how information is encoded in any possible configuration state of information. The holographic principle tells us that information is encoded on an event horizon that arises in the accelerated frame of reference of an observer. The event horizon is a bounding surface of space that surrounds the observer. A bit of information is like a yes/no question that can only be answered yes or no, like the question: Is a particle located at this position in space or not? Relativity theory tells us the absolute nature of space cannot be parameterized in terms of an absolute coordinate system that localizes positions in space. Quantum theory tells us that there is always some uncertainty in the position coordinates of space. Only the relative frame of reference of an observer can be parameterized in terms of a coordinate system, but there is always some uncertainty in the way that parameterization occurs. The amount of uncertainty in the position coordinates is called the Planck length. This uncertainty is inherent in an uncertainty relationship, $\Delta x \Delta y \geq \ell^2$, that describes how position coordinates on a bounding surface of space can be measured. A position coordinate is smeared out and acts like a pixel on the surface that encodes a bit of information.

Information is encoded in a non-commutative geometry because the position coordinates on the surface are smeared out into pixels. This parameterization is in terms of matrices. Each eigenvalue of the matrix localizes a position on the surface, but the coordinate is smeared out into a pixel of size ℓ^2. An SU(2) matrix encodes information in a binary code, like a spin ½ variable that can only point up or down. The spin ½ variable is like an on/off switch that encodes information in a binary code of 1's and 0's. If n position coordinates are defined on the bounding surface, the parameterization is as an nxn matrix. An SU(n) matrix can always be decomposed into SU(2) matrices, and so information for the n position coordinates can always be decomposed into n bits of information. This is the natural way information becomes encoded on a bounding surface of space in a non-commutative geometry. Each pixel on the screen encodes a quantized bit of information, but that information is entangled due to the nature of matrices.

The holographic principle is a duality that relates the point particle description of a world to the holographic screen description. These are equivalent descriptions of a world. In other words, there are two equivalent ways to formulate the laws of the universe and determine the quantum state of potentiality for the universe.

The point particle formulation is about the behavior of matter and energy within 3+1 dimensional space and time. The holographic screen formulation describes the behavior of anything that can appear to happen in some three dimensional region of space in terms of bits of information encoded on the two dimensional bounding surface of that space. Our modern understanding of quantum information tells us that the holographic screen description is more fundamental than the point particle description, since all the fundamental quantized bits of information for anything we can observe in a three dimensional region of space are encoded on the two dimensional bounding surface of that space. Each pixel on the screen encodes a fundamental quantized bit of information.

The reason the two formulations are equivalent to each other is because each bit of information is like a yes/no question that asks: Is a particle observable at this position in space or not? A holographic screen is a bounding surface of space that encodes information for everything that can appear to happen in the three dimensional region of space bounded by that two dimensional surface. A bit of information encoded on the screen specifies whether a particle can be observed at some point in the three dimensional space bounded by the screen at some moment of time. A sequence of yes/no questions over a sequence of events leads to the observation of the particle located at some point in space at each moment of time. The observation of the particle as it follows its trajectory through space over the course of time is no different than an animation of images on a digital computer screen over a sequence of screen outputs. Each screen output is an observational event. Like a computer animation, the pixel size on the screen sets a limit on how accurately we can resolve things in space. Within that animation of events, the particle can appear to follow a trajectory through space over the course of time.

When we formulate the quantum state of potentiality in terms of the 3+1 dimensional point particle description, we describe the behavior of particles that follow trajectories through space over the course of time. When we formulate the quantum state in terms of the holographic screen description, we describe configuration states of information on the screen and how those configuration states are updated over a sequence of screen outputs. Each screen output is an event, and the flow of time arises as a sequence of events in the flow of energy. The two descriptions are aspects of a duality and are equivalent to each other.

The fundamental reason these two descriptions are equivalent to each other is due to the nature of the observer. It is the observer that ties the two descriptions together and allows them to be equivalent aspects of a duality. The holographic screen is only an event horizon that arises in the accelerated frame of reference of the observer. The observer's accelerated frame of reference is characterized by the observer's time-like worldline through that space-time geometry. The observer's worldline is only an ordered sequence of observational events that arise in the flow of energy. Inherent in the notion of a worldline is the concept of proper time, which gives a measure of the geometrical length of the worldline through the space-time geometry. The concept of an action principle in terms of kinetic and potential energy arises from proper time, and the path of least action represents the shortest distance between two points in the geometry.

With each event on its worldline, the observer observes another screen output from its screen. The images of things observed by the observer are projected from its screen to the central point of view of the observer with each screen output. The images are animated over a sequence of events, just like images on a digital computer screen. All that really happens is the configuration state of the screen is updated over a sequence of screen outputs. All the images of things displayed on the screen are projected to the central point of view of the observer.

The holographic principle is a duality relating the holographic screen description of the world to a point particle description. Just like anything else the observer can observe in its world, point particles are no more real than animated images displayed on a holographic screen, arising from the way bits of information are encoded on the screen over a sequence of screen outputs.

The point particle description of the world is usually formulated as a quantum field theory. A QFT always assumes the existence of a background space-time geometry. That background space-time geometry must exist before any particles can become observable. Point particles can only exist within a pre-existing space and time. Relativity theory tells us that all space-time geometries are relative to each other, and there is no such thing as a pre-existing space and time. This is the ultimate chicken and egg problem. How can point particles really exist if there is no such thing as a pre-existing space and time within which they can exist? The answer is they only appear to exist within an observer's frame of reference.

A direct corollary of this answer is that before the observer can enter into a frame of reference, before any particles can appear to exist in the observer's world, the observer must exist. The solution to the chicken and egg problem is that only an observer has a pre-existing existence. Only an observer has an existence that pre-exists anything that appears to come into existence in the observer's world. All observable things are observer-dependent, as their observation depends on the frame of reference of the observer. Taken to its logical conclusion, before anything is created in the observer's world in a big bang event, nothing exists.

The point particle description of the world is usually formulated as a QFT defined in Minkowski space. Since this formulation has Lorenz invariance and Poincare symmetry under translation and rotation in the space-time geometry, it is possible to define a vacuum state, and then define particle excitations from the vacuum. All observers in Minkowski space will agree on particle excitations from the vacuum state. This is only possible if a vacuum state can be defined. Mach's principle tells us that fundamentally the vacuum state cannot be defined.

Unfortunately, once we introduce gravity into the equation and an event horizon arises for an observer in an accelerated frame of reference, Poincare symmetry is no longer in effect, and different observers will no longer agree on the nature of particle excitations. An observer in a freely falling frame of reference does not observe the same particle excitations as an observer in an accelerated frame of reference, which leads to the paradoxes of Hawking radiation. These paradoxes fundamentally have to do with the freedom the observer has to enter into any possible frame of reference, including a freely falling frame of reference in which particles of Hawking radiation simply disappear from existence. What appears to exist for an accelerated observer is not the same as for a freely falling observer.

The pre-existing space-time geometry assumed in any quantum field theory is equivalent to the frame of reference of an observer. Any space-time geometry assumed in a QFT is equivalent to an observer's frame of reference. Any QFT constrains the nature of the observer's frame of reference, which is why a QFT is not consistent with relativity theory. The holographic principle allows the observer to enter into any possible frame of reference. The observer always has the freedom to enter into a freely falling frame of reference.

Quantum Electrodynamics, like most QFTs, assumes a background space-time geometry of flat Minkowski space. QED describes the electromagnetic force, or the interaction of photons with charged particles like the electron. The predictions of QED have been experimentally confirmed to an astounding 11 significant figures, but QED cannot describe the behavior of photons in a curved space-time geometry, like the space-time geometry that characterizes the gravitational field of a black hole. The reason for this deficiency is that QED cannot describe the observations of an observer near the event horizon of a black hole. Event horizons naturally arise with the force of gravity, but no QFT that assumes the existence of a pre-existing space-time geometry can fully encompass gravity.

Gravity allows for all possible space-time geometries and the associated event horizons that arise with the force of gravity. Fundamentally, these space-time geometries are all relative to each other and arise in the accelerated frame of reference of an observer. The differing observations of observers that observe things in different frames of reference can only be described in the differing kinds of space-time geometries that arise in the accelerated frames of reference of different observers. It is precisely this inability of any QFT to fully encompass gravity that led to the discovery of the holographic principle. The holographic principle allows the observer to enter into any possible frame of reference.

A QFT can never fully encompass the holographic principle since it tries to force the observer into a particular frame of reference, while the holographic principle allows the observer to enter into any possible frame of reference. Any space-time geometry assumed in any QFT is equivalent to an observer's frame of reference.

The problem with any quantum field theory as a description of the world is a QFT assumes a single space-time geometry observed by multiple observers, and so assumes that multiple observers have the same fame of reference. This is the assumption of a single world observed by multiple observers. This assumption is false. Every observer has its own world defined by its own frame of reference, which defines the space-time geometry of that world. This does not rule out the possibility of a consensual reality shared by many different observers if their differing frames of reference are able to share information.

If a QFT assumes a background space-time geometry of flat Minkowski space, there is no force of gravity, and all observers have the same frame of reference except for the possibility of uniform relative motion. If a QFT assumes a curved space-time geometry, like that arising from a black hole, all observers have the same accelerated frame of reference and observe the effects of the same event horizon. The problem with these assumptions is they do not fully encompass the nature of gravity. A freely falling observer does not observe the same effects of an event horizon observed by an observer in an accelerated frame of reference that is equivalent to the force of gravity. With differing accelerated frames of reference, different observers observe differing effects of gravity. Even worse is the situation that arises in cosmology with the exponential expansion of space. Every observer in an exponentially expanding space makes observations in a bounded space that is limited by the observer's own cosmic horizon. There is no possible way any QFT can fully encompass all of these situations.

The point particle description of the world is not wrong, but is only an aspect of the holographic duality. With an accelerated frame of reference an event horizon arises that acts as a holographic screen and encodes information. Like any other observable object in space, particle images are projected from the screen to the central point of view of the observer. In reality, only the observer can enter into an accelerated fame of reference. In reality, only the observer has an existence that pre-exists whatever appears to come into existence in the observer's world.

Due to the magic of holography, a 2+1 dimensional holographic screen description of the world appears as a 3+1 dimensional point particle description of the world. That is the nature of the duality. Once we understand the nature of the duality, it is easier to understand the quantum theory formulation of particle physics. The quantum field theory formulation of particle physics is horrendously complicated, while the holographic screen formulation is very simple.

The two key ideas that reduce the complexity of particle physics are the central role played by the observer and the fundamental nature of quantum information. We only have to keep in mind that only an observer can enter into an accelerated frame of reference, which allows a holographic screen to arise and an animated sequence of screen outputs to become displayed. Whatever the observer happens to observe on those screen outputs, we are only talking about the way information is encoded on the screen as a configuration state of information.

The holographic animation of images projected from the screen to the observer arises in a sequence of events defined on the observer's accelerated worldline, which is exactly the same as a sequence of screen outputs. Those configuration states are inherent in any quantum state of potentiality. The big mystery is the process of choice that chooses a particular configuration state from the quantum state of potentiality with each screen output.

All the mystery about quantum theory and all the debate about the correct interpretation of quantum theory comes down to this mystery about how choices are made. In classical physics there is no choice, since only the path of least action can be followed. In quantum physics there is a choice, and there are many possible paths that can be followed. A path that deviates from the path of least action in some sense is like a path that bends the laws of physics, like the law of gravity. We usually don't think of it this way, but when gravity is combined with quantum theory, the law of gravity can be bent. The classical way we understand the law of gravity is in terms of the path of least action, but quantum theory allows for paths that deviate from the classical path. To speak in metaphorical terms, if the law of gravity is bent enough, it is possible to walk on water.

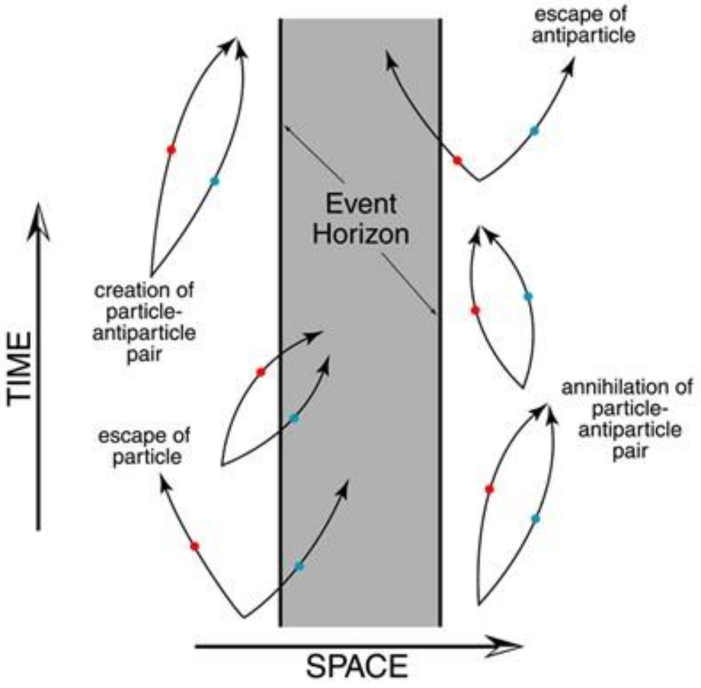
The fundamental way quantum theory formulates any quantum state of potentiality is in terms of an action principle and a sum over all possible paths. Action is defined in terms of the geometrical length of the path in the geometry, and the sum is over all possible paths in the geometry. Each path is weighted with a probability factor that we call the wave function. This kind of formulation is easiest to understand for point particle motion, but also applies to the motion of more complicated geometrical objects, like strings and membranes. The remarkable thing is that when these kinds of geometrical objects are considered, the holographic principle naturally emerges from the mathematical analysis [4].

The holographic principle is a duality that mathematically relates the observable behavior of objects in space to the information encoded on a holographic screen. Observable objects in space can appear as point particles, strings, membranes, or more complex geometrical objects that arise as bound states of simpler geometrical objects, but fundamentally, those objects in space reduce down to the way bits of information are encoded on a holographic screen.

The correct way to interpret relativity theory is in terms of the pre-existing existence of the observer, which leads directly to the holographic principle. It is not so much that the point particle formulation of quantum theory is wrong, but that it is not the whole story. The nature of a hologram is there are always two sides to any story. This state of affairs is called a duality in theoretical physics, and the holographic principle is the most fundamental duality known about in physics. The point particle formulation of physical reality is still an accurate description of our observations, but that formulation is dual to the holographic screen formulation. We do indeed observe the behavior of particles that appear to exist within some kind of pre-existing space and time, but all the information for that behavior is encoded on a holographic screen. The screen only arises because the observer enters into an accelerated frame of reference.

The holographic screen encodes quantized bits of information for the behavior of anything that appears to exist in the space bounded by that bounding surface. Perceivable things appear as three dimensional objects in space, but the bits of information are encoded on a two dimensional bounding surface. The bounding surface is only an event horizon that arises in the accelerated frame of reference of an observer. In reality, only the observer has a pre-existing existence. That existence cannot stop existing, but must have an underlying reality.

Another way to understand the encoding of information on the surface of an event horizon, like the event horizon of a black hole, is in terms of the separation of virtual particle-antiparticle pairs at the horizon. The basic idea is quantum uncertainty allows for the virtual creation of particle-antiparticle pairs everywhere in empty space. Those virtual pairs are spontaneously created out of nothing and normally annihilate back into nothing in a short period of time as specified by the uncertainty principle, but at an event horizon something very odd appears to happen as observed by an external observer. The virtual pairs can appear to separate from each other at the horizon. The virtual particle that is radiated away from the horizon towards the external observer appears to become a real particle. The encoding of information on the horizon is directly related to this separation of virtual particle-antiparticle pairs at the horizon [10].



Energy conservation requires the virtual particle to carry an equal but opposite amount of energy as that carried by the antiparticle, so that the total energy of the virtual process adds up to zero. The particle that is radiated away from the horizon towards the external observer carries positive energy or heat, while the antiparticle that falls across the horizon into the black hole carries negative energy and decreases the mass of the black hole. From the perspective of the external observer, the event horizon of the black hole radiates away positive energy thermal radiation, which is called Hawking radiation.

This thermal radiation allows us to calculate the temperature of the horizon. The smallest amount of energy that can be radiated away from the event horizon is a photon with an energy specified by its frequency or wavelength as $E=hf=hc/\lambda$.

Quantum uncertainty specifies that this smallest energy photon has a wavelength about equal to the maximal circumference of the horizon, $\lambda=2\pi R$, where R is the radius of the horizon. Again, this makes use of the constraint on the wavelength that it must fit into the circumference of a periodic orbit, which is the requirement of periodicity in a compact space. This wavelength corresponds to a photon that is just barely gravitationally bound to the black hole. As observed by the external observer, the event horizon demarcates a boundary in space where the force of gravity is so strong that even light cannot escape away from the black hole. Only a photon that is just barely bound to the black hole can escape away.

Relativity theory tells us the radius of the event horizon is determined by the mass of the black hole as $R=2GM/c^2$. The Planck length is the smallest possible distance scale, and corresponds to the smallest possible mass, which is called the Planck mass. It is a simple matter to determine the radius of a Planck size black hole by equating the energy of the smallest energy photon gravitationally bound to the black hole $E=hc/\lambda=\hbar c/R$ with $E=Mc^2$, and setting $\ell=R/\sqrt{2}$, this gives the Planck length as $\ell^2=\hbar G/c^3$.

In terms of the holographic principle, the energy of this smallest energy photon corresponds to the thermal energy of the smallest possible bit of information encoded on the surface of the horizon. In the sense of thermodynamics, this smallest possible bit of information is the smallest possible degree of freedom, and carries an amount of thermal energy $E=kT$. Fundamentally, this thermal energy arises as a bit of information encoded in a binary code of 1's and 0's tends to flip back and forth, just like a switch that tends to flip back and forth between the on and the off positions due to its thermal energy. This thermal energy represents the kinetic energy of the bits of information. If we equate the thermal energy of the fundamental bits of information with the energy carried by the smallest energy photon radiated away from the horizon, we find $kT=\hbar c/R$.

This thermal radiation leads to an infamous paradox. If the surrounding space is cold enough, the black hole will radiate away thermal radiation and decrease in mass until the black hole evaporates away and disappears. What happens to all the information for all the things that fell into the black hole? Does all that information disappear as the black hole evaporates away or does it come out with the Hawking radiation? How can all the information for the things that fell into the black hole come out with the Hawking radiation since no signal that originates inside the black hole can ever cross the horizon?

The answer to this puzzle is called horizon complementarity, and directly follows from the principle of equivalence. From the perspective of an external observer, nothing ever really fell into the black hole in the first place. As things appear to fall into the black hole, those things appear to stop right at the event horizon due to gravitational infrared Doppler shifting, which is a kind of time dilation. For the external observer, it takes an infinite amount of time for anything to cross the event horizon and fall into the black hole. From that external perspective, those things stop right at the horizon and all the information becomes scrambled. That scrambled information is then radiated back to the external observer as Hawking radiation, and so no information is ever lost as the black hole evaporates away.

But things appear to be very different for a freely falling observer that falls right through the event horizon. The horizon only appears to exist for the external observer. For the freely falling observer the event horizon is just an imaginary surface in space and does not really exist. For the freely falling observer there is no event horizon and there are no particles of Hawking radiation.

For the external observer the event horizon is a real surface in space that appears to have a temperature and to radiate away a form of thermal radiation called Hawking radiation towards the external observer, but for the freely falling observer there is no event horizon and there are no thermal particles of Hawking radiation. For the freely falling observer these things simply do not exist. They only appear to exist from the accelerated perspective of the external observer.

How can what appears to exist be so radically different for the external observer and for the freely falling observer? The principle of equivalence gives the answer, and is called horizon complementarity. The external observer is in an accelerated frame of reference. That acceleration is equivalent to the force of gravity that the external observer perceives to arise from the black hole. That force of gravity is equivalent to the external observer's accelerated frame of reference. Only in that accelerated frame of reference does the external observer perceive the event horizon of the black hole and the associated particles of Hawking radiation that are radiated away from the horizon towards the external observer. In the freely falling frame of reference of the freely falling observer, there is no force of gravity, there is no event horizon, and there are no particles of Hawking radiation.

There is no real paradox here because it is impossible for the external observer to compare its observations with the observations of the freely falling observer. The absolute impossibility of ever comparing their radically different observations is called horizon complementarity. As long as it remains absolutely impossible for different observers to compare their radically different observations about what appears to exist, there is no real disagreement in those observations.

The holographic principle is a duality that relates the point particle description of what appears to happen in a region of three dimensional space to the information encoded on the two dimensional bounding surface of that space. Particles of Hawking radiation appear to move in that three dimensional space, but all the information for that behavior is encoded on the bounding surface of that space, which acts as a holographic screen. Observable images of particles are projected from the screen to the external observer and are animated over a sequence of events, just like images animated on a computer screen. The holographic screen is an event horizon that arises as the observer enters into an accelerated frame of reference and follows a time-like worldline. In reality, only the observer moves.

To be clear about things, the holographic principle is only about what appears to exist in a three dimensional region of space. The appearance of things in space is like the projection of images from a holographic screen to the central point of view of an observer. Those images are animated over a sequence of events as the observer follows an accelerated worldline. Everything that appears to exist in that region of space is reducible to bits of information encoded on the bounding surface of that space, which acts as a holographic screen. The screen can only arise if the observer of the screen enters into an accelerated frame of reference.

All perceivable objects in space, from elementary particles to macroscopic bodies, are like images that are projected from a holographic screen to the central point of view of an observer. The perceivable images are all observer-dependent, as they can appear to come into existence or disappear from existence depending on the frame of reference of the observer.

The observer's holographic screen is characterized by a quantum state of potentiality that describes all possible ways in which information can become encoded on the screen. The quantum state is only a sum over all possible configuration states of information, which is the same as a sum over all possible paths in the geometry. A path in the geometry is an observer's worldline. Every event on the observer's worldline is a decision point where the quantum state branches into all possible paths, and there is always a choice about which path to follow.

A choice is a decision point that chooses one particular path, or one particular configuration state of information each event, which is called a quantum state reduction in quantum theory. The observation of that particular configuration state of information is like a screen output from the observer's holographic screen, just like a screen output from a digital computer screen. The most likely path in the sense of quantum probability is the path of least action, which is like the path that expends the least amount of energy, or the shortest distance path that connects two events on the observer's worldline. We understand this as the principle of least action, where action represents proper time or the geometrical length of the worldline.

There are many different ways in which information can become encoded on the screen. To be precise, if there are n pixels that encode information in a binary code, there are $N=2^n$ different ways to encode that information. The sum over all possible ways in which the information can become encoded is called a quantum state of potentiality. Each distinct way in which information can become encoded defines an actual state of information that can be observed. The quantum state of potentiality is only a sum over all possible actual states. Each actual state is an observable state. In some sense, observable states are like a screen output from the holographic screen. In any observation, an observable state is actually observed. The quantum state of potentiality characterizing the holographic screen is only a sum over all possible actual states. An actual state is a possible observable state, like a screen output from the holographic screen.

How do these screen outputs arise? To be clear about things, the holographic screen is only a bounding surface of space that encodes information. A bounding surface is only an event horizon that arises in the accelerated frame of reference of an observer, and limits the observer's observation within that bounded space. A bounding surface only arises because the observer enters into an accelerated frame of reference. The nature of that accelerated frame of reference is the expenditure of energy, just like a rocket ship that must expend energy through its thrusters as it accelerates through space. An observer in an accelerated frame of reference expends energy equivalent to a force, and an event horizon arises.

An observer in an accelerated frame of reference follows a path through space and time. That path is called a worldline. An observer that follows an accelerated worldline expends energy on its path. In some sense, every screen output from the holographic screen corresponds to another observable event on that worldline.

In the sense of animation, a sequence of screen outputs arising over a sequence of events is just like the animation of a movie as displayed on a digital computer screen. That animated sequence of events always arises in the flow of energy that characterizes the observer's accelerated worldline. Images projected from the screen are animated over a sequence of screen outputs.

Quantum theory tells us every event is a decision point where the quantum state of potentiality branches into all possible paths, and there is always a choice about which path to follow. The quantum state of potentiality for a holographic screen is only a sum over all possible ways information can become encoded on the screen. Each distinct way of encoding information on the screen defines an actual or observable state, or a configuration state of information.

The quantum state is only a sum over all possible configuration states. Each possible configuration state is a possible screen output. With any observation, a particular configuration state is actually observed. With each screen output there is always a choice about which particular configuration state to observe, and which particular path to follow.

The configuration states can only arise in the accelerated frame of reference of an observer since that is the only way a holographic screen arises. Every event on the observer's accelerated worldline is a decision point about which path to follow and which configuration state of information to observe. The observer's worldline is a possible path through the geometry.

There is a big puzzle in quantum theory about how a particular configuration state or path is chosen from the quantum state in any particular observation. The quantum state of potentiality is only a sum over all possible configuration states. In any observation of something a particular configuration state must be chosen from the quantum state. The way quantum theory addresses this problem is to assign a probability factor to each configuration state. This probability factor is the essence of the wave function $\psi=e^{i\theta}$, where the phase angle θ is related to the action as $\theta=S/\hbar$, and the action ΔS is directly proportional to the proper time $\Delta\tau$, which measures the geometrical length of the worldline. The wave function is a probability wave. The way this probability wave is calculated is called the action principle. The details of the action principle do not concern us here except to say the amount of action is like the amount of energy expended on any path.

The maximal probability for anything to actually happen is to follow the path of least action, which is like following the path that expends the least amount of energy. Quantum theory expresses this state of affairs in terms of quantum probability. The most likely path in the sense of quantum probability is the path of least action, which is the path that expends the least energy.

In the sense of relativity theory, the path of least action is like the shortest distance between two points in a curved space-time geometry, like the path of a great circle that connects two points on the surface of a sphere. Following the shortest distance path between two points in the geometry expends the least amount of energy. The fundamental way relativity theory and quantum theory are connected is only possible because all energy arises in the accelerated frame of reference of an observer. The amount of action that characterizes any possible path is written in terms of kinetic and potential energy as $\Delta S=\Delta t[KE-PE]$. At a fundamental level, kinetic energy arises with all motion from the effects of time dilation, while potential energy arises in an accelerated frame of reference from the effects of accelerated motion.

The quantum state of potentiality is only a sum over all possible paths. Each path is weighted with a probability factor that depends on the amount of action that characterizes that path. The amount of action is like the amount of energy that must be expended to follow that path. Fundamentally, this arises as the observer enters into an accelerated frame of reference and follows a time-like worldline. The observer's worldline is a frame of reference that is always defined in the context of a space-time geometry. The amount of action that characterizes any possible worldline is directly related to the geometrical length of that worldline as defined in that space-time geometry. The geometrical length of the worldline is called the proper time and is related to the metric as $\tau=\int ds$, where $ds^2=g_{\mu\nu}dx^\mu dx^\nu$. The integration is along the length of the worldline. The metric only measures the curvature of the space-time geometry in that particular frame of reference. The metric can only take on a specific form in a specific space-time geometry. The Kaluza-Klein mechanism with six extra compactified dimensions of space defined at each point of the usual 3+1 extended dimensions of the space-time geometry extends this idea of a gravitational metric to include all four fundamental forces. A cosmological constant and the exponential expansion of space extends this idea into the realm of cosmology.

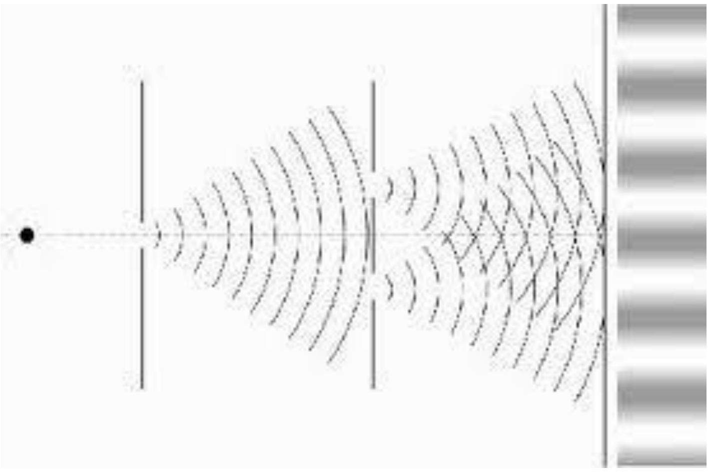
The holographic principle gives a translation of the metric formulation of physics, as it describes whatever appears to happen in some three dimensional extended region of space over the course of time, into the holographic screen formulation of physics, as it describes how configuration states of information are encoded on the two dimensional bounding surface of that extended region of space over a sequence of screen outputs. Each screen output is an event on the observer's worldline. This is only a translation from the point particle formulation of physics into the holographic screen formulation. The Kaluza-Klein mechanism extends physics into extra compactified dimensions of space, and allows for the internal structure of particles. The discovery of the secrets of this translation for the kind of space-time geometry describing the universe is still a work in progress [11], but there can be no doubt that there is such a translation.

Even when the secrets of this translation are finally discovered, this will not give a definitive formulation for the behavior of the universe. There is no such thing as a definitive formulation for the behavior of the universe. This bears repeating:

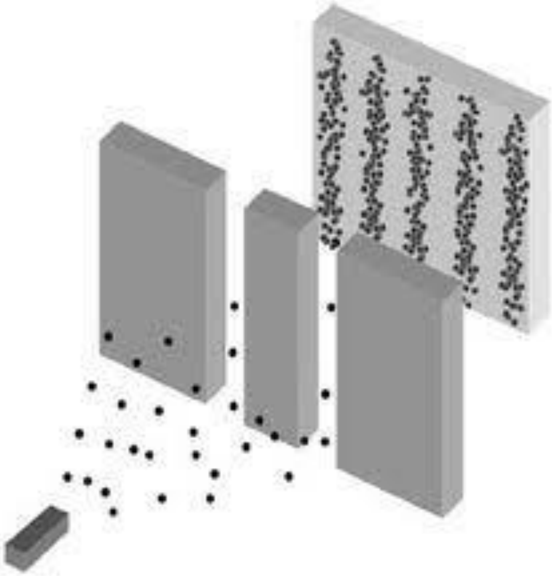
There is no such thing as a definitive formulation for the behavior of the universe.

There is no definitive formulation for the behavior of any observer's world since we can never constrain the observer's frame of reference. The way a holographic screen arises and information is encoded on the screen depends on the frame of reference of the observer. The observer is always free to enter into any possible frame of reference, including a freely falling frame of reference in which the bounding surface of space and all the information that describes things in the observer's world simply disappear. The observer's freedom to enter into any possible frame of reference is the fundamental reason for the one-world-per-observer paradigm of modern cosmology, as we'll see in the next section. Simply stated, in an ultimate freely falling frame of reference, the observer's world disappears and the observer no longer has a world. In a fundamental way, this state of affairs arises from the inherent uncertainty of all measurements in the observer's world, including the measurement of space itself.

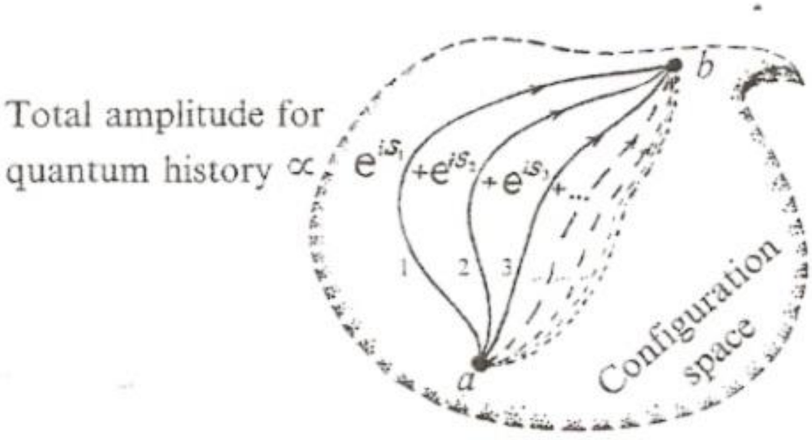
There is one aspect of the action principle that is worth a closer look. The probability factors of quantum theory are probability waves. The probability factors determine the probability with which any path can be followed, which depends on the amount of action that characterizes that path. The amount of action for a path is like the amount of energy expended to follow that path. The probability factors act like waves, and tend to add together when waves in phase with each other and tend to cancel out when waves are out of phase with each other. The result of this wave addition and cancellation is an interference pattern, as seen in the double slit experiment [12].



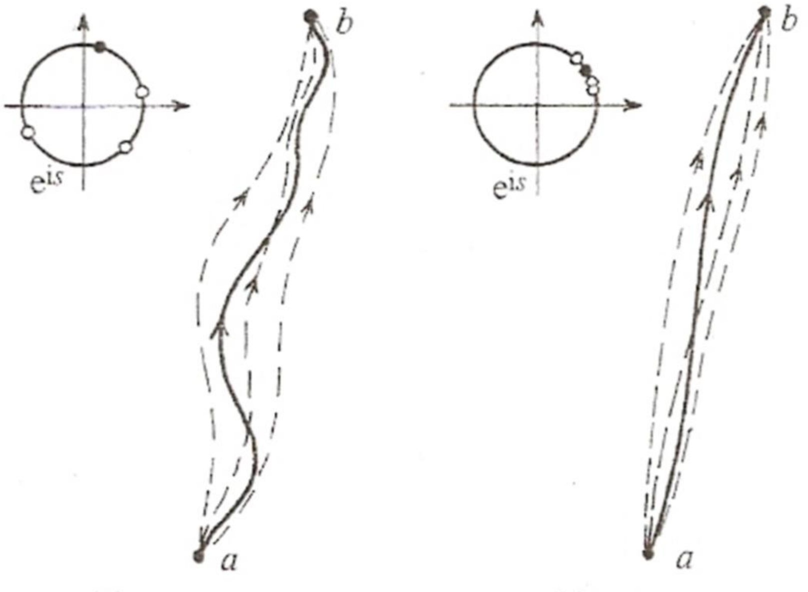
What exactly do the probability factors represent? They give the probability for measurement. In the double slit experiment, they give probability that a photon can be measured at a particular point in space at a particular moment of time. The photon is a quantum of electromagnetic radiation and acts like a particle, with a location in space. When we measure that position in space at a moment of time, we measure a particle-like property of the photon. When we measure the interference pattern, we measure a wave-like property. The interference pattern arises from the probability waves that characterize the photon [13].



The quantum state of potentiality is only a sum over all the probability factors that correspond to all possible paths. Each path is weighted with a probability factor called the wave function $\psi=e^{i\theta}$. In this sum, the probability waves tend to cancel out when they are out of phase with each other, but add together and reinforce each other when they are in phase with each other. This gives rise to the path of least action as the path that gives the maximum probability [9].



This is easiest to see in the complex plane, where the probability factors act like vectors defined on the unit circle, $z=e^{i\theta}=x+iy$, where $x=\cos\theta$ and $y=\sin\theta$. Since the unit vectors are oriented relative to the x-axis with a phase angle θ, they tend to cancel out when they point in random uncorrelated directions, but tend to add together when they align together and point in the same or correlated directions. The principle of least action follows from the fact that the maximum correlation occurs at a stationary point, which corresponds to minimizing the action [9].



(a) (b)

The interference pattern that results from this correlation of information is the basis for holography. The interference pattern is how information is encoded on the two dimensional surface of a hologram, which allows for the projection of three dimensional images from the hologram. The encoding of information fundamentally arises from the correlated nature of the wave-interference nature of quantum theory. This is not only the case for ordinary holograms, but also for the holographic principle, which extends this correlation to space-time geometries.

Quantum theory only tells us how to calculate the quantum state of potentiality and assign a probability to each possible configuration state. The quantum state is expressed as a sum over all possible paths, or a sum over all configuration states. The probability factors arise from the geometrical length of the path, and the most likely path is like the shortest distance between two points in the geometry. In the sense of the holographic principle, the observer’s path is defined by the accelerated worldline of the observer. Each event on that path is like a screen output from the observer’s holographic screen that chooses a particular configuration state.

How are these choices made? Quantum theory says the choices are made randomly. With every observational event or screen output, a configuration state is randomly chosen from the quantum state. This process of random choice randomly measures the quantum state of potentiality, which is like a probability distribution. Since the path of least action is the most likely path in the sense of quantum probability, as long as the choices are made randomly, the path that is most likely followed is the path of least action.

What about the possibility of biased choice? There is nothing in quantum theory or the laws of physics that rules out the possibility of biased choice. Quantum theory and the laws of physics only determine a quantum state of potentiality. Any process of choice, whether that process is random or biased in nature, is always outside the laws of physics. The possibility of biased choice no more violates the laws of physics than random choice violates the laws of physics.

If choices are made randomly, the path of least action is most likely followed and the laws of physics have predictability. On the other hand, if choices are made in a biased way, the laws of physics lose their predictability. Since physicists don't like unpredictability, they have arbitrarily ruled out the possibility of biased choice, but there is no good reason in either the laws of physics or in quantum theory, other than unpredictability, to do so.

Any process of choice we can postulate is outside the laws of physics since the laws only determine a quantum state of potentiality. The quantum state is always determined through an action principle and a sum over all possible paths. The laws of physics are inherent in the action principle. Every decision point on the path is a choice about which branch of the path to follow. Whatever process of choice we postulate, that process is always outside the laws of physics.

This is a humongous problem in the conceptual formation of quantum theory that theoretical physicists have not been able to solve. The fundamental reason they have not been able to solve it is because a process of choice is always outside the laws of physics and cannot be formulated. Everything that can ever appear to happen in the observer’s world is included in the quantum state of potentiality. The laws of physics only determine the quantum state. In the observation of any actual thing, a choice must be made as an actual configuration state is chosen from the quantum state, and a particular branch of the path is actually followed.

What determines this process of choice? The answer is nobody knows. We are now out of the realm of physics and into the realm of metaphysics. Actually, the answer is not that difficult. Only the observer can make a choice just as only the observer can make an observation of something or enter into a relative frame of reference in which that thing appears. Only the observer can choose the things that it observes, but what is the true nature of the observer?

The observer is nothing more than the consciousness present at the center of its world. The consciousness is not defined by anything it perceives in its world, and neither is the observer's process of choice. This is the mystery that we will never be able to scientifically explain. Only the observer can observe things in its world, and only the observer can choose which things to observe. An observer's choice about which things to observe is called the focus of attention of consciousness. Like consciousness itself, the focus of attention is an unexplainable mystery.

An observer's choice about what to observe in its world expresses the bias inherent in this process of choice. How is this bias expressed? Fundamentally, the bias arises from the nature of the flow of energy in the observer's world. As the flow of energy comes into alignment, feelings of connection are perceived. Those feelings of connection feel good, and so the observer is naturally biased to choose feelings of connection over feelings of disconnection, which feel bad and arise as the flow of energy goes out of alignment. This bias to choose feelings of connection tends to keep the flow of energy in alignment. This bias to choose good feelings and avoid bad feelings arises with the observer's biased focus of attention on its world.

Quantum theory tells us that as long as choices are made in an unbiased way, things tend to follow the path of least action, which is the most energy efficient way for things to act since it expends the least amount of energy. This tendency with unbiased choice for things to follow the path of least action arises from the U(1) invariant form of the probability factors $\psi=e^{i\theta}$. This tendency to follow the path of least action is inextricably connected to the space-time geometry of the observer's world through the geometrical length of the observer's path through its world.

As things follow the path of least action, the flow of energy through the observer's world tends to come into alignment. This is the normal way for energy to flow through the observer's world. As long as the flow of energy through things in the observer's world remain in alignment, the observer will perceive feelings of connection that arise from that alignment.

Feelings of connection feel good while feelings of disconnection feel bad, so the observer is naturally biased to choose feelings of connection over disconnection, which tends to bring the observer's choices into alignment with the normal flow of things that naturally arises with unbiased choices. As long as the observer's choices stay in alignment with the normal flow of things, aligned actions are naturally expressed and feelings of connection are perceived.

This natural bias to choose feelings of connection allows energy to flow in the normal way, and leads to the best of all possible worlds. This is really the only way the flow of energy in the observer's world can come into alignment and feelings of connection can be perceived. All biased expressions that interfere with the normal flow of things in the observer’s world can only lead to feelings of disconnection that feel bad and make things worse.

Why would an observer ever become biased to choose feelings of disconnection and interfere with the normal flow of things in the observer's world? The only way to answer this question is to discuss the strange phenomena of self-identification. The observer is nothing more that the consciousness present at the center of its world, but that world is always organized around a central form of information we call a body. The observer's body is only a bound state of information in its world that tends to coherently self-replicate form, but for reasons we'll discuss in a later section, the observer tends to identify itself with that central form of information.

Once this self-identification occurs, the observer becomes biased to defend the survival of that central form of information as though the observer's existence depends on it. In a very real sense, self-defensive expressions arise with the fear of non-existence. Once the observer becomes biased to defend itself, the observer then feels compelled to choose feelings of disconnection and interfere with the normal flow of things in its world for the purposes of self-defense. Once this kind of biased choice is expressed, events become unpredictable in the observer's world.

There are only two natural ways in which choices can be made and bias can be expressed. Either an observer chooses to feel connected and brings itself into alignment with the normal flow of things in its world, or an observer chooses to defend itself. Satisfaction of desires arise in the context of feelings of connection. Even the expression of curiosity and creativity, or the appreciation of creative expressions, only arise in the context of feelings of connection. On the other hand, self-defensive expressions can only arise in the context of the fear of non-existence. The fear of non-existence always underlies self-defense and feelings of disconnection.

The last important topic for this section is the nature of coherent organization. Coherent organization of information is the very nature of holography, arising from the wave-interference nature of quantum theory. The holographic principle relies on coherence in just this way. At a fundamental level, there is a tendency for bits of information defined on a bounding surface of space to align together and thereby form coherently organized bound states of information. The images of the objects in space projected from the holographic screen to the central point of view of the observer can only arise with the formation of bound states of information.

Fundamentally, this tendency for the alignment of bits of information arises in much the same way the flow of energy comes into alignment, as a reflection of symmetry. The principle of least action arises from the U(1) symmetry of the wave function $\psi=e^{i\theta}$. The tendency for bits of information to align together on a bounding surface of space arises from the SU(n) symmetry inherent in how n bits of information are encoded on the bounding surface. Since the bits of information are defined by the eigenvalues of an SU(n) matrix, this information is entangled together in the sense of quantum entanglement. We know from quantum theory that entangled bits of information tend to align together, and thereby form larger and larger bound states of information over the course of time.

It helps to back up a bit and discuss the course of time from the big bang event. We'll continue this discussion in the next section, but for now let's focus on the development of coherent organization. As the observer's world expands in size from the big bang event and energy flows through the observer's world, bits of information tend to become organized into coherently organized forms that hold together and self-replicate form over a sequence of events. These coherently organized forms are the nature of images projected from the holographic screen. Coherently organized forms develop due to the tendency of bits of information on the screen to align with each other, just like entangled spin variables.

Coherently organized forms become disorganized due to the tendency of bits of information to lose that alignment, which is a consequence of the thermal energy they carry as they tend to flip back and forth. We quantify that thermal energy in terms of temperature as E=kT, and understand that thermal energy is random kinetic energy. We also understand that only potential energy can give rise to the tendency for bound states to form. Fundamentally, potential energy represents the tendency for bits of information to align together as a consequence of the entanglement of information that arises with matrices. This always occurs in the context of attractive forces, or in an observer's accelerated frame of reference.

There is always a thermodynamic balance between the tendency for bits of information to form coherently organized bound states due to the interactions between bits of information that tend to bring them into alignment and the tendency for coherently organized forms to become disorganized due to the thermal energy each bit of information carries as it tends to flip back and forth. Entanglement tends to align and organize, while thermal energy tends to disorganize.

Coherent organization develops through phase transitions, like the freezing of liquid water into ice, and is lost through phase transitions, like the melting of ice into liquid water. Biological processes are a lot more complicated than physical processes in that a process of eating allows for reproduction of form in addition to self-replication of form, but otherwise are similar in that they are driven by the same flow of energy. The normal flow of energy through the observer's world drives all thermodynamic processes, allowing coherently organized forms to develop and self-replicate their form, but then eventually become disorganized and lose their form. In the process, the distinct observable images of things appear to come into existence, appear to exist for a brief period of time, and then appear to go out of existence.

The best way to understand the nature of the perceivable images is as bound states of information. Bound states have a quality called coherent organization, which means they tend to self-replicate their forms over a sequence of events. The self-replication of coherently organized bound states is a thermodynamic property, like a frozen piece of ice composed of water molecules that appears to be the same object in space from moment to moment even though the water molecules constantly move around in space. The piece of ice is a coherently organized bound state of water molecules that tends to self-replicate its form over a sequence of event. This coherent binding is a thermodynamic property. When the ice melts into liquid water the coherent organization is lost, called a phase transition, and the form of the water molecules changes.

In a very similar way, the distinct images of things in space projected from the holographic screen are coherently organized bound states of information that tend to self-replicate their forms over a sequence of events. Bits of information are encoded on the screen in a binary code of 1′s and 0′s, like a spin ½ variable that can only point up or down, or an on/off switch. Due to their thermal energy the bits of information tend to switch back and forth, but due to their entangled interactions they tend to align together and form bound states over a sequence of events.

The easiest way to understand the interactions and the formation of bound states is with a spin network[9]. Bits of information encoded on a bounding surface of space are very much like a spin network. Each pixel on the screen encodes a bit of information like a spin ½ variable. As spin variables interact with each other, they tend to become entangled. Bits of information defined in terms of the eigenvalues of an SU(n) matrix are entangled. Due to their entanglement, interacting spin variables tend to align together and form larger and larger bound states over a sequence of events, since there is a greater combinatorial probability for alignment than non-alignment. For example, two entangled states, each composed of two spin ½ variables with total spin zero, and each represented as an entangled state $\Psi=|\uparrow>|\downarrow>-|\downarrow>|\uparrow>$, can combine into a spin 0 state with probability ¼ or into a spin 1 state with probability ¾.

The size of the bound states that form is only limited by thermodynamics. The tendency to form larger coherently organized bound states, as entangled bits of information align together, is opposed by the tendency for entropy to increase and bound states to become disorganized over time, as bits of information flip back and forth due to their thermal energy. This tendency for entropy to increase, or for heat to flow from a hotter to colder object, is fundamentally related to the encoding of information on a holographic screen as we'll see in the next section.

Due to these opposing tendencies for organization and disorganization to develop, all distinct bound states only have a temporary apparent existence as they appear to come into existence and then go out of existence. The brief period of time they appear to exist arises from their tendency to coherently self-replicate their forms.

Coherently organized forms that self-replicate form are animated in the flow of energy that characterizes the observer's world. That flow of energy only arises as the observer enters into an accelerated frame of reference. As we'll see in the next section, the ultimate source of that flow of energy is the exponential expansion of space itself, which drives the expansion of the observer's world from the big bang event.

The holographic principle describes the world of an observer in terms of the encoding of bits of information on a bounding surface of space that arises as the observer enters into an accelerated frame of reference. This formulation describes what appears to happen in the three dimensional space bounded by a two dimensional bounding surface of space. Holography is deeply ingrained in the geometrical nature of relativity theory and the wave-interference nature of quantum theory.

There are other attempts to describe the world that do not rely on holography, but instead use the geometrical idea of the world as defined on a three dimensional hyper-sphere that is imbedded in a four dimensional hyper-space. These kinds of formulations always place the observer at the center of the three dimensional hyper-sphere, and so are very similar in nature to the holographic approach, but to my knowledge, they cannot explain the disappearance of the world through the mechanism of horizon complementarity. As we'll see in a later section, it is the disappearance of the observer's world that connects holography to non-dual metaphysics.

This section began with a quote from Plato, and ends with a bookend quote from Nisargadatta Maharaj:

> *You know yourself only through the senses and the mind. You take yourself to be what they suggest; having no direct knowledge of yourself. You have mere ideas. Whatever you think you are you take it to be true-imagining yourself perceivable.*
> *I see as you see, hear as you hear.*
> *All this I perceive quite clearly, but I am not in it.*
> *I feel myself as floating over it, aloof and detached. There is also the awareness of it all and a sense of immense distance as if the body and the mind and all that happens to them were somewhere far out on the horizon.*
> *I am like a cinema screen-clear and empty. The pictures pass over it and disappear, leaving it as clear and empty as before. In no way is the screen affected by the pictures, nor are the pictures affected by the screen.*
> *The screen intercepts and reflects the pictures. These are lumps of destiny, but not my destiny; the destinies of the people on the screen.*
> *The character will become a person when he begins to shape his life instead of accepting it as it comes-identifying himself with it.*
> *To myself I am neither perceivable nor conceivable; there is nothing I can point out and say "this I am".*

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